1987: PART A

1) $a(t)=6t-18,\ v(0)=24,\ x(1)=20$

a) $v(t)=\int a(t)\,dt$

$v(t)=\int (6t-18)\,dt$

$v(t)=3t^2-18t+C$

$v(0)=24$

$24=C$

$v(t)=3t^2-18t+24$

b) At rest when $v(t)=0$

$3t^2-18t+24=0$

$t^2-6t+8=0$

$(t-2)(t-4)=0$

$t=2,\ t=4$

c) $x(t)=\int (3t^2-18t+24)\,dt$

$x(t)=t^3-9t^2+24t+C$

$x(1)=20$

$20=1-9+24+C$

$C=4$

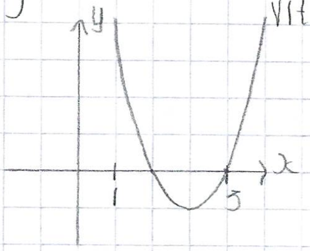
$x(t)=t^3-9t^2+24t+4$

d) Total Distance $=\int_1^3 |v(t)|\,dt$ or $\int_1^2 v(t)\,dt-\int_2^3 v(t)\,dt$

$v(t)=3t^2-18t+24$

$=3(t^2-6t+8)$

$=3(t-2)(t-4)$



$=\int_1^2 (3t^2-18t+24)\,dt-\int_2^3 (3t^2-18t+2$

$=4-(-2)$

$=4+2$

$=6$

2) $f(x)=\sqrt{1-\sin x}$

a) $1-\sin x\geq 0$

$-1<\sin x<1$

$\therefore$ $1-\sin x$ is always greater than or equal to 0

Domain: All reals

b) $f'(x)=\frac{1}{2}(1-\sin x)^{-1/2}(-\cos x)$

$=-\dfrac{\cos x}{2\sqrt{1-\sin x}}$

c) $1-\sin x\neq 0$

$\sin x\neq 1$

$x\neq \frac{\pi}{2}+2\pi n$

a) $f(0) = \sqrt{1-\sin 0} = 1$

$f'(0) = \dfrac{-\cos 0}{2\sqrt{1-\sin 0}} = -\dfrac{1}{2}$

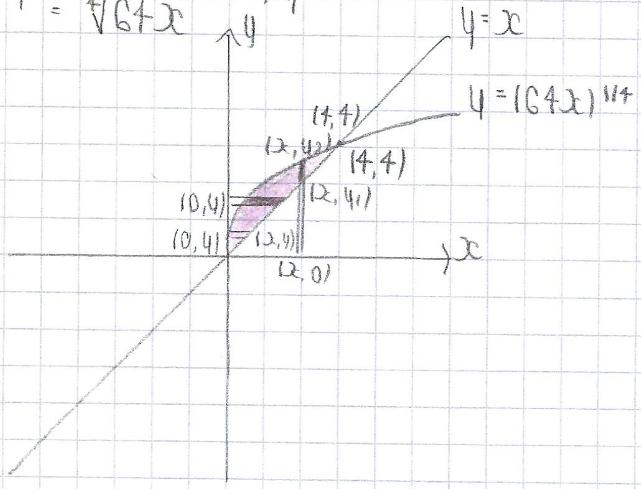
Equ. of tangent:

$y - 1 = -\frac{1}{2}(x-0)$

$y = -\frac{1}{2}x + 1$

3) $y = (64x)^{1/4}$, $y = x$

$= \sqrt[4]{64x}$



$(64x)^{1/4} = x$

$64x = x^4$

$x^3 = 64$

$x = 64^{1/3} = 4$

a) $R(x) = y_2 - 0 = (64x)^{1/4}$

$r(x) = y_1 - 0 = x$

$\text{Volume} = \pi\int_0^4 (((64x)^{1/4})^2 - x^2)\,dx$

$= \pi\int_0^4 ((64x)^{1/2} - x^2)\,dx = \pi\left[\frac{8x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^4 = \pi\left(\frac{2}{3}(8)(8) - \frac{64}{3} - 0\right) = \frac{64}{3}\pi$

b) $R(x) =$

$r(x) =$

$f(x) = 2\ln(x^2+3) - x \qquad -3 \le x \le 5$

a) $f'(x) = 2\left(\frac{2x}{x^2+3}\right) - 1 = \frac{4x}{x^2+3} - 1$

$f'(x) = 0$

$\frac{4x}{x^2+3} - 1 = 0$

$\frac{4x}{x^2+3} = 1$

$\frac{4x}{x^2+3} - 1 = 0$

$\frac{4x - (x^2+3)}{x^2+3} = 0$

$\frac{-x^2+4x-3}{x^2+3} = 0$

$-x^2+4x-3 = 0 \quad , \quad x^2+3=0$

$x^2 - 4x + 3 = 0 \quad , \quad x^2 \neq -3$

$(x-1)(x-3) = 0$

$x = 1, x = 3$

$f'(x)$: -3 ($-$) 1 ($+$) 3 ($-$) 5

Rel. Max at $x = 3$ since $f'(x)$ changes sign from +ve to −ve @ $x = 3$. Rel. min at $x = 1$ since $f'(x)$ changes sign from −ve to +ve

b) $f''(x) = \frac{(x^2+3)(4) - 4x(2x)}{(x^2+3)^2} = \frac{4x^2+12-8x^2}{(x^2+3)^2} = \frac{-4x^2+12}{(x^2+3)^2} = 0$

$f''(x) = 0$

$\frac{-4x^2+12}{(x^2+3)^2} = 0$

$-4x^2+12 = 0 \qquad (x^2+3)^2 = 0$

$4x^2 = 12 \qquad x^2+3 \neq 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$f''(x)$: -3 ($-$, $\cap$) $\sqrt{3}$ ($+$, $\cup$) $\sqrt{3}$ ($-$, $\cap$) 5

Pt. of inflection at $x = -\sqrt{3}, x = \sqrt{3}$

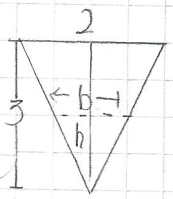
c) $f(3) = 2\ln(12) - 3 = 1.970$

Check endpoint extrema:

$f(-3) = 2\ln 12 + 3 = 7.970$

$f(5) = 2\ln 28 - 5 = -0.030$

Abs. max value $= 2\ln 12 + 3 = 7.970$ to 3.d.p.

5) 

a) Volume $= \frac{1}{2}(2)(3)(5) = \frac{30}{2} = 15 \ ft^3$

b) when $V = \frac{1}{4}\left(\frac{30}{2}\right) = \frac{30}{8} = \frac{15}{4}$, we want $\frac{db}{dt}$

By similar triangles: $\frac{2}{3} = \frac{b}{h}$

$2h = 3b$

$b = \frac{2h}{3}$

$V = \frac{1}{2}bh(5)$

$= \frac{1}{2}\left(\frac{2}{3}h\right)h(5)$

$= \frac{5}{3}h^2$

$\frac{dV}{dt} = \frac{10}{3}h\frac{dh}{dt}$

$-2 = \frac{10}{3}\left(\frac{3}{2}\right)\frac{dh}{dt}$

$-2 = \frac{30}{6}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-12}{30} = -\frac{4}{10} = -\frac{2}{5}$ ft/min

$\frac{1}{2}bh(5) = \frac{15}{4}$

$\frac{1}{2}\left(\frac{2}{3}h\right)h = \frac{15}{20}$

$h^2 = \frac{15}{20} \cdot \frac{6}{2} = \frac{90}{40} = \frac{9}{4}$

$h = \frac{3}{2}$

c) We want $\frac{dA}{dt}$, when $V = \frac{15}{4}$ or $h = \frac{3}{2}$

$A = 5b$

$A = 5\left(\frac{2}{3}h\right)$

$A = \frac{10}{3}h$

$\frac{dA}{dt} = \frac{10}{3}\frac{dh}{dt}$

$= \frac{10}{3}\left(-\frac{2}{5}\right)$

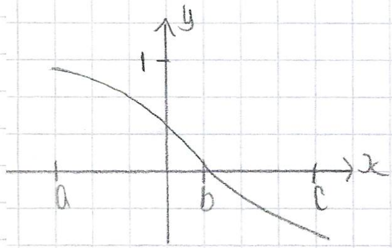
$= -\frac{20}{15}$

$= -\frac{4}{3}$ ft^2/min

6) $f(x) < 1$, $f'(x) < 0$

a) $f(b) = 0$, $a < b < c$

Since $f'(x) < 0 \Rightarrow f$ is strictly decreasing on $[a, c]$



Area $= \int_a^b f(x)\,dx - \int_b^c f(x)\,dx$

b) $g(x) = \frac{1}{f(x)-1} = (f(x)-1)^{-1} \Rightarrow g'(x) = -[f(x)-1]^{-2}(f'(x))$

Since $f(x) < 1$, $-[f(x)-1]^{-2} < 0$, and $f'(x) < 0$, $g'(x) > 0$ $\therefore$ g is increas

c) $h'(x) < 0$

$F(x) = h(f(x)) \Rightarrow F'(x) = h'(f(x)) \cdot f'(x)$

Since $h'(x) < 0$ and $f'(x) < 0$, $F'(x) > 0 \therefore F$ is increasin